

Dear Rising 8th Graders,

Congratulations! I look forward to teaching you this upcoming fall. Attached you will find the 8th grade Carlisle mathematics summer work.

We will be covering a rigorous Algebra I curriculum this upcoming school year. If you are seeking additional math work this summer (other than the enclosed work) I recommend purchasing an ALEKS subscription for the months of July/August and taking the course RLT8.

Please make sure to have a strong foundational knowledge of your Pre-Algebra skills which includes: number sense overall, integers, fractions, multi-step equations, scientific notation, unit rates, ratios and proportional relationships to name a few. I will collect the enclosed math work on the 1st day of school.

I hope you have a wonderful summer and I look forward to working with you this upcoming school year.

Mrs. Spiegel

# Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
  - Parentheses/Brackets (inside to outside)
  - Exponents
  - Multiplication/Division (left to right)
  - Addition/Subtraction (left to right)

ex:  $9x^2 - 4(y + 3z)$   
for  $x = -3, y = 2, z = 5$

$$9(-3)^2 - 4(2 + 3 \cdot 5)$$

$$9(-3)^2 - 4(2 + 15)$$

$$9(-3)^2 - 4 \cdot 17$$

$$9 \cdot 9 - 4 \cdot 17$$

$$81 - 4 \cdot 17$$

$$81 - 68 = \boxed{13}$$

# The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex:  $5(8x - 3)$

$$5(8x - 3)$$

$$5(8x) - 5(3)$$

$$\boxed{40x - 15}$$

# Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex:  $2(3x - 4) - 12x + 9$

$$2(3x - 4) - 12x + 9$$

$$6x - 8 - 12x + 9$$

$$\boxed{-6x + 1}$$

Evaluate each expression for  $a = 9$ ,  $b = -3$ ,  $c = -2$ ,  $d = 7$ . Show your work.

1. $a - cd$	2. $2b^3 + c^2$	3. $\frac{a + d - c}{b}$	4. $(a - b)^2 + d(a + c)$
5. $4c - (b - a)$	6. $\frac{a}{b} - 5a$	7. $2bc + d(12 - 5)$	8. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$	10. $7(y + 3)$	11. $-3(4w - 3)$	12. $(6r + 3)2$
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Simplify each expression, showing all work.

13. $8(x + 1) - 12x$	14. $6w - 7 + 12w - 3z$	15. $9n - 8 + 3(2n - 11)$	16. $3(7x + 4y) - 2(2x + y)$
17. $(15 + 8d)(-5) - 24d + d$	18. $9(b - 1) - c + 3b + c$	19. $20f - 4(5f + 4) + 16$	20. $8(h - 4) - h - (h + 7)$

# Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex:  $-18 = 6j$

$$\frac{-18}{6} = \frac{6j}{6}$$

$$-3 = j \rightarrow \boxed{j = -3}$$

# Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex:  $\frac{a}{7} - 12 = -9$

$$\frac{a}{7} - 12 = -9$$
$$+12 \quad +12$$

$$\frac{a}{7} = 3$$
$$\times 7 \quad \times 7$$

$$\boxed{a = 21}$$

# Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex:  $5(2x - 1) = 3x + 4x - 1$

$$10x - 5 = 3x + 4x - 1$$

$$10x - 5 = 7x - 1$$
$$-7x \quad -7x$$

$$3x - 5 = -1$$

$$+5 \quad +5$$

$$3x = 4$$
$$\div 3 \quad \div 3$$

$$\boxed{x = \frac{4}{3}}$$

Solve each equation, showing all work.

21.  $f - 64 = -23$

22.  $-7 = 2d$

23.  $\frac{b}{-12} = -6$

24.  $13 = m + 21$

25.  $5x - 3 = -28$

26.  $\frac{w + 8}{-3} = -9$

27.  $-8 + \frac{h}{4} = 13$

28.  $22 = 6y + 7$

29.  $8x - 4 = 3x + 1$

30.  $-2(5d - 8) = 20$

31.  $7r + 21 = 49r$

32.  $-9g - 3 = -3(3g + 2)$

33.  $5(3x - 2) = 5(4x + 1)$

34.  $3d - 4 + d = 8d - (-12)$

35.  $f - 6 = -2f + 3(f - 2)$

36.  $-2(y - 1) = 4y - (y + 2)$

# Scientific Notation

Standard Form to Scientific Notation: move the decimal after the first non-zero digit and eliminate any trailing zeros. Multiply by 10 to the power equal to the number of places you moved the decimal point. If the original number was greater than 1, the exponent is positive. If the number was less than 1, the exponent is negative.

ex: 0.0000571

$$0.\underline{00005}71$$

Original number < 1, so negative exponent

$$= \boxed{5.71 \times 10^{-5}}$$

Scientific Notation to Standard Form: move the decimal point the number of places indicated by the exponent. If the exponent is positive, move the decimal right. If negative, move left.

ex:  $3.5 \times 10^3$

Positive exponent, so move decimal right

$$3.\underline{500} = \boxed{3,500}$$

# Negative Exponents & Simplifying Monomials

Zero Exponent: Any number raised to the zero power equals 1

ex:  $y^0 = 1$

Negative Exponent: Move the base to the opposite side of the fraction line and make the exponent positive

ex:  $x^{-4} = \frac{1}{x^4}$

Monomial x Monomial: Multiply the coefficients and add the exponents of like bases

ex:  $(4x^3)(2x^5) = 8x^8$

Monomial ÷ Monomial: Divide the coefficients and subtract the exponents of like bases

ex:  $\frac{a}{a^6} = a^{-5} = \frac{1}{a^5}$

Power of a Monomial: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex:  $(-2fg^5)^3 = -8f^3g^{15}$

Power of a Quotient: Raise each base (including the coefficient) to that power. If a base already has an exponent, multiply the two exponents

ex:  $\left(\frac{5d^3}{c}\right)^2 = \frac{25d^6}{c^2}$

Convert each number to Scientific Notation.

37. 67,000,000,000	38. 0.0009213	39. 0.00000000004	40. 3,201,000,000,000,000
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Convert each number to Standard Form.

41. $5.92 \times 10^{-5}$	42. $1.1 \times 10^7$	43. $6.733 \times 10^{-8}$	44. $3.27 \times 10^2$
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Simplify each expression. Write your answers using only positive exponents.

45. $w^{-9}$	46. $\frac{m^5}{m^2}$	47. $f^5 \cdot f^3$	48. $\left(\frac{h^2}{g}\right)^3$
49. $(a^5)^2$	50. $\frac{1}{b^{-3}}$	51. $z^0$	52. $4r^6 \cdot 3r \cdot 2r^2$
53. $\frac{qp^{-2}}{3q^{-3}}$	54. $\frac{8d^3}{2cd^{-2}}$	55. $(g^4h)^2 \cdot (2g^3h^{-1})^2$	56. $(6a)^0$
57. $(-3n^2k^4)^2$	58. $\left(\frac{w^5x^{-2}y}{w^2xy^4}\right)^3$	59. $\frac{6 \cdot 10^7}{2 \cdot 10^3}$	60. $(1.5 \cdot 10^{-6}) \cdot (4 \cdot 10^9)$

# Solving Proportions

1. Set the two cross-products equal to each other
2. Solve the equation for the variable

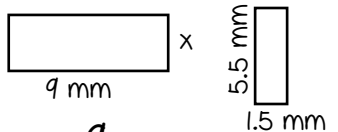
ex:  $\frac{m}{4} = \frac{3}{5}$

$$\frac{5m}{5} = \frac{12}{5}$$

$$m = 2.4$$

# Similar Figures

1. To find a missing side length, set up a proportion, matching up corresponding sides.
2. Solve the proportion using the steps above.

ex: 

$$\frac{x}{1.5} = \frac{9}{5.5}$$

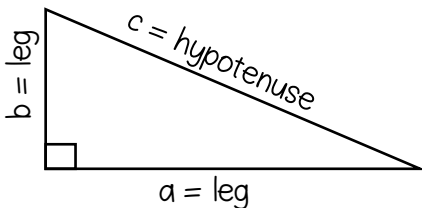
$$x = 2.45 \text{ mm}$$

# The Pythagorean Theorem

\*\*\* The Pythagorean Theorem applies to right triangles only \*\*

The sides next to the right angle (a & b) are legs

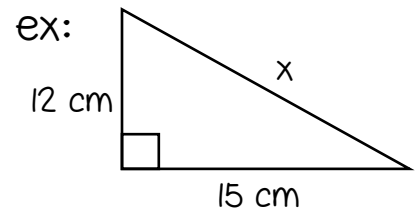
The side across from the right angle (c) is the hypotenuse



Pythagorean Theorem:  $a^2 + b^2 = c^2$

To find the hypotenuse: add the squares of the legs and then find the square root of the sum

To find a leg: subtract the square of the given leg from the square of the hypotenuse and then find the square root of the difference



x is the hypotenuse

$$12^2 + 15^2 = x^2$$

$$144 + 225 = x^2$$

$$369 = x^2$$

$$x = \sqrt{369} \approx 19.2 \text{ cm}$$

ex:  $a = ?$ ,  $b = 3$ ,  $c = 6$

a is a leg

$$a^2 + 3^2 = 6^2$$

$$a^2 + 9 = 36$$

$$a^2 = 36 - 9 = 27$$

$$a = \sqrt{27} \approx 5.2$$



Solve each proportion, showing all work.

76.  $\frac{6}{7} = \frac{4}{m}$

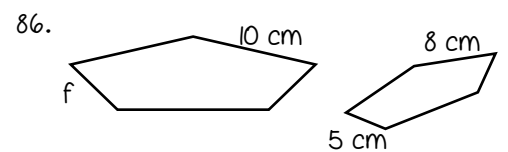
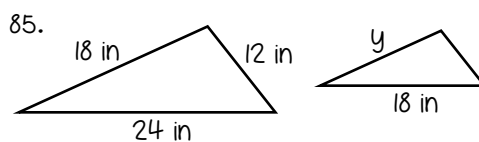
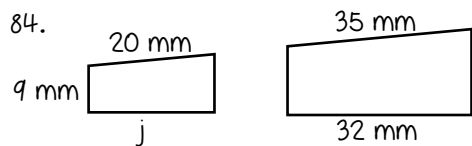
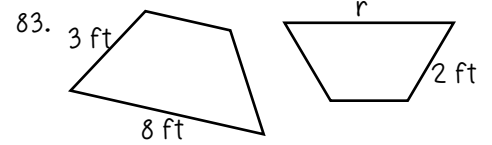
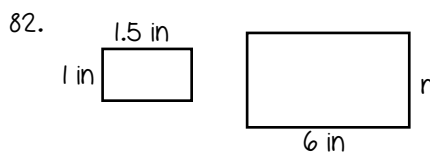
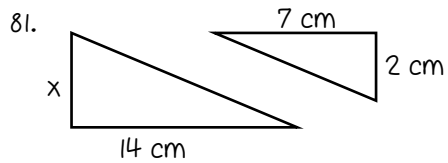
77.  $\frac{12}{5} = \frac{k}{3}$

78.  $\frac{h}{7} = \frac{8}{2}$

79.  $\frac{22}{n} = \frac{9}{36}$

80.  $\frac{4}{21} = \frac{3}{c}$

Assume each pair of figures is similar. Find the missing side length, showing all work.



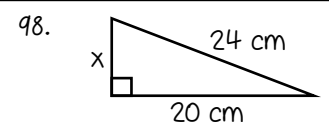
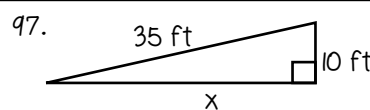
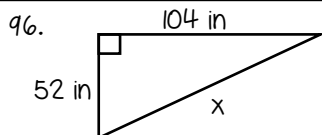
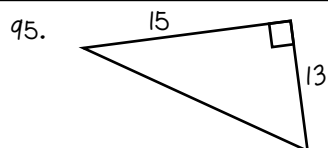
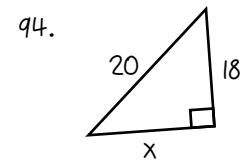
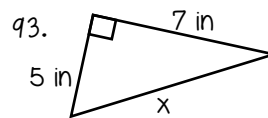
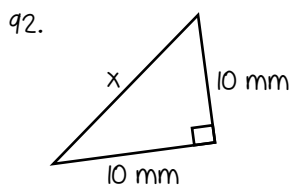
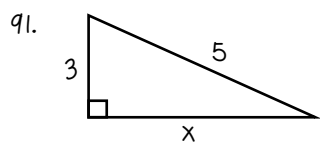
Find the missing side length in each right triangle to the nearest tenth. Show your work!

87.  $a = 6, b = 8, c = ?$

88.  $a = ?, b = 9 \text{ cm}, c = 13 \text{ cm}$

89.  $a = 7, b = ?, c = 14$

90.  $a = 14 \text{ in}, b = 14 \text{ in}, c = ?$



Determine whether or not you can form a right triangle from the given side lengths. Explain.

99. 18, 22, 26

100. 5, 12, 13

Evaluate each expression for  $a = 9$ ,  $b = -3$ ,  $c = -2$ ,  $d = 7$ . Show your work.

1. $a - cd$ $23$	2. $2b^3 + c^2$ $-50$	3. $\frac{a + d - c}{b}$ $-6$	4. $(a - b)^2 + d(a + c)$ $193$
5. $4c - (b - a)$ $4$	6. $\frac{a}{b} - 5a$ $-48$	7. $2bc + d(12 - 5)$ $61$	8. $b + 0.5[8 - (2c + a)]$ $-1.5$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$ $10g - 40$	10. $7(y + 3)$ $7y + 21$	11. $-3(4w - 3)$ $-12w + 9$	12. $(6r + 3)2$ $12r + 6$
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Simplify each expression, showing all work.

13. $8(x + 1) - 12x$ $-4x + 8$	14. $6w - 7 + 12w - 3z$ $18w - 3z - 7$	15. $9n - 8 + 3(2n - 11)$ $15n - 41$	16. $3(7x + 4y) - 2(2x + y)$ $17x + 10y$
17. $(15 + 8d)(-5) - 24d + d$ $-63d - 75$	18. $9(b - 1) - c + 3b + c$ $12b - 9$	19. $20f - 4(5f + 4) + 16$ $0$	20. $8(h - 4) - h - (h + 7)$ $6h - 39$

Solve each equation, showing all work.

21.  $f - 64 = -23$

$$f = 41$$

22.  $-7 = 2d$

$$d = -7/2 = -3.5$$

23.  $\frac{b}{-12} = -6$

$$b = 72$$

24.  $13 = m + 21$

$$m = -8$$

25.  $5x - 3 = -28$

$$x = -5$$

26.  $\frac{w + 8}{-3} = -9$

$$w = 19$$

27.  $-8 + \frac{h}{4} = 13$

$$h = 84$$

28.  $22 = 6y + 7$

$$y = 5/2 = 2.5$$

29.  $8x - 4 = 3x + 1$

$$x = 1$$

30.  $-2(5d - 8) = 20$

$$x = -2/5 = -0.4$$

31.  $7r + 21 = 49r$

$$r = 1/2 = 0.5$$

32.  $-9g - 3 = -3(3g + 2)$

no solution

33.  $5(3x - 2) = 5(4x + 1)$

$$x = -3$$

34.  $3d - 4 + d = 8d - (-12)$

$$d = -4$$

35.  $f - 6 = -2f + 3(f - 2)$

all real numbers

36.  $-2(y - 1) = 4y - (y + 2)$

$$y = 4/5 = 0.8$$

Convert each number to Scientific Notation.

37. 67,000,000,000 $6.7 \times 10^{10}$	38. 0.0009213 $9.213 \times 10^{-4}$	39. 0.00000000004 $4 \times 10^{-11}$	40. 3,201,000,000,000,000 $3.201 \times 10^{15}$
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Convert each number to Standard Form.

41. $5.92 \times 10^{-5}$ 0.0000592	42. $1.1 \times 10^7$ 11,000,000	43. $6.733 \times 10^{-8}$ 0.00000006733	44. $3.27 \times 10^2$ 327
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Simplify each expression. Write your answers using only positive exponents.

45. $w^{-9}$ $\frac{1}{w^9}$	46. $\frac{m^5}{m^2}$ $m^3$	47. $f^5 \cdot f^3$ $f^8$	48. $\left(\frac{h^2}{g}\right)^3$ $\frac{h^6}{g^3}$
49. $(a^5)^2$ $a^{10}$	50. $\frac{1}{b^{-3}}$ $b^3$	51. $z^0$ 1	52. $4r^6 \cdot 3r \cdot 2r^2$ $24r^9$
53. $\frac{qp^{-2}}{3q^{-3}}$ $\frac{3q^3}{p^2}$	54. $\frac{8d^3}{2cd^{-2}}$ $\frac{4d^5}{c}$	55. $(g^4h)^2 \cdot (2g^3h^{-1})^2$ $4g^{14}$	56. $(6a)^0$ 1
57. $(-3n^2k^4)^2$ $9n^4k^8$	58. $\left(\frac{w^5x^{-2}y}{w^2xy^4}\right)^3$ $\frac{w^9}{x^9y^9}$	59. $\frac{6 \cdot 10^7}{2 \cdot 10^3}$ $3 \cdot 10^4$	60. $(1.5 \cdot 10^{-6}) \cdot (4 \cdot 10^9)$ $6 \cdot 10^3$

Solve each proportion, showing all work.

$$76. \frac{6}{7} = \frac{4}{m}$$

$$m = 4.\bar{6}$$

$$77. \frac{12}{5} = \frac{k}{3}$$

$$k = 7.2$$

$$78. \frac{h}{7} = \frac{8}{2}$$

$$h = 28$$

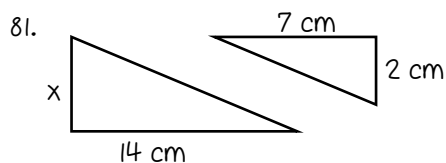
$$79. \frac{22}{n} = \frac{9}{36}$$

$$h = 88$$

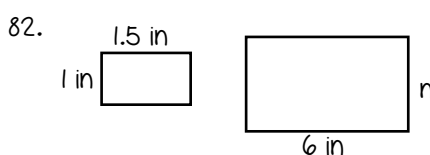
$$80. \frac{4}{21} = \frac{3}{c}$$

$$h = 15.75$$

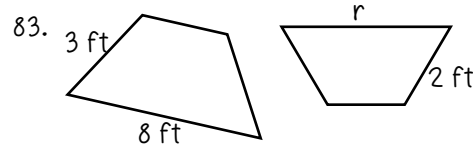
Assume each pair of figures is similar. Find the missing side length to the nearest tenth.



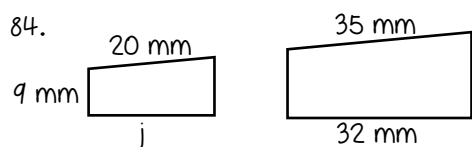
$$x = 4 \text{ cm}$$



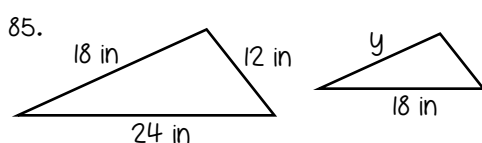
$$x = 4 \text{ in}$$



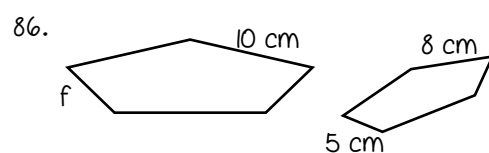
$$x = 5.\bar{3} \text{ ft}$$



$$j \approx 18.3 \text{ mm}$$



$$y = 13.5 \text{ in}$$



$$f = 6.25 \text{ cm}$$

Find the missing side length in each right triangle to the nearest tenth. Show your work!

87.  $a = 6, b = 8, c = ?$

$$c = 10 \text{ units}$$

88.  $a = ?, b = 9 \text{ cm}, c = 13 \text{ cm}$

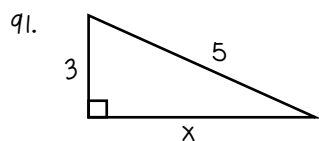
$$a \approx 9.4 \text{ cm}$$

89.  $a = 7, b = ?, c = 14$

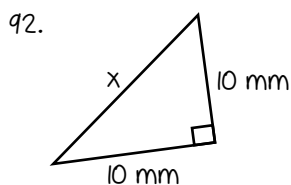
$$b \approx 12.1 \text{ units}$$

90.  $a = 14 \text{ in}, b = 14 \text{ in}, c = ?$

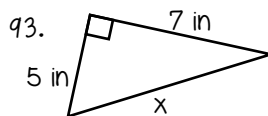
$$c \approx 19.8 \text{ in}$$



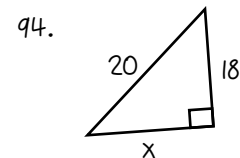
$$x = 4 \text{ units}$$



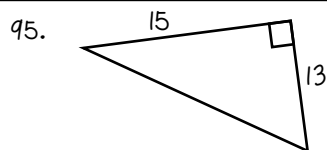
$$x \approx 14.1 \text{ mm}$$



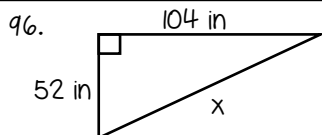
$$x \approx 8.6 \text{ in}$$



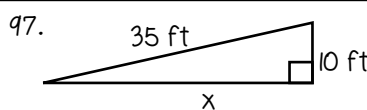
$$x \approx 8.7 \text{ units}$$



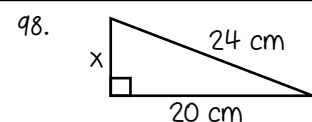
$$x \approx 19.8 \text{ units}$$



$$x \approx 116.3 \text{ in}$$



$$x \approx 33.5 \text{ ft}$$



$$x \approx 13.3 \text{ cm}$$

Determine whether or not you can form a right triangle from the given side lengths. Explain.

99. 18, 22, 26

$$\text{No; } 18^2 + 22^2 \neq 26^2$$

100. 5, 12, 13

$$\text{Yes; } 5^2 + 12^2 = 13^2$$

Solve the proportion.

77.  $\frac{h}{6} = \frac{20}{24}$

78.  $\frac{5}{7} = \frac{c}{14}$

79.  $\frac{6}{8} = \frac{21}{b}$

80.  $\frac{30}{j} = \frac{26}{39}$

81.  $\frac{5}{k} = \frac{15}{20}$

82.  $\frac{32}{112} = \frac{a}{14}$

83.  $\frac{16}{7} = \frac{18}{g}$

84.  $\frac{w}{60} = \frac{15}{200}$

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Solve the percent problem.

85. Find 15% of 85.

86. 6 is 75% of what number?

87. 40 is what percent of 320?

88. What is 20% of 45?

89. 70 is what percent of 350?

90. Find  $33.\bar{3}\%$  of 81.

91. A \$58 camera is on sale for 20% off. Find the sale price.

92. Find the total price of a \$14.00 shirt including the 7% sales tax.

# Geometry

## Geometry Basics

- Perimeter is the distance around a polygon
- Circumference is the distance around a circle
- Area is the space inside a figure
- Volume is the capacity of a 3-dimensional figure
- Surface Area is the sum of the areas of all the faces on a 3-dimensional figure

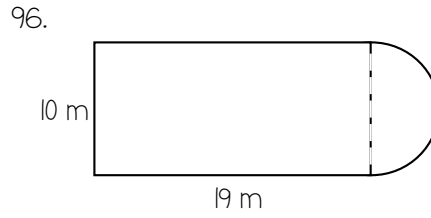
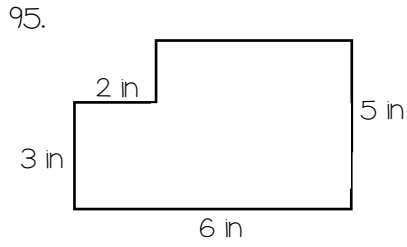
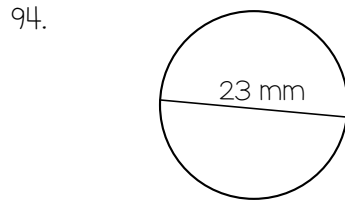
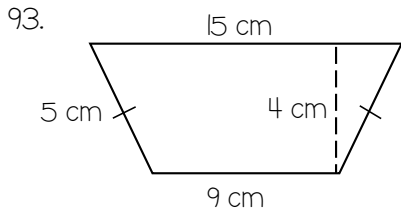
## 2-Dimensional Geometry Formulas

- Perimeter of Any Figure: sum of side lengths
- Circumference =  $\pi \cdot \text{diameter}$
- Area of Parallelogram = base  $\cdot$  height
- Area of Triangle =  $\frac{1}{2} \cdot \text{base} \cdot \text{height}$
- Area of Trapezoid =  $\frac{1}{2} \cdot \text{height}(\text{base}_1 + \text{base}_2)$
- Area of Circle =  $\pi \cdot \text{radius}^2$

## 3-Dimensional Geometry Formulas

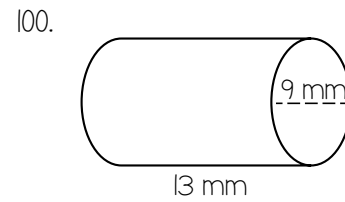
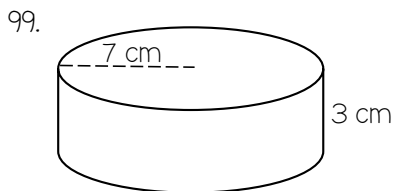
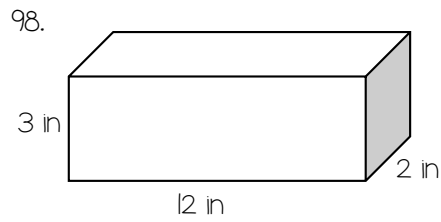
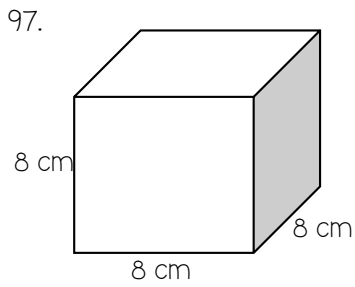
- Volume of Rectangular Prism = length  $\cdot$  width  $\cdot$  height
- Volume of Cylinder =  $\pi \cdot \text{radius}^2 \cdot \text{height}$
- Surface Area of Rectangular Prism =  $2 \cdot \text{length} \cdot \text{width} + 2 \cdot \text{length} \cdot \text{height} + 2 \cdot \text{height} \cdot \text{width}$
- Surface Area of Cylinder =  $2 \cdot \pi \cdot \text{radius}^2 + 2 \cdot \pi \cdot \text{radius} \cdot \text{height}$

Find the perimeter (or circumference) and area. Use 3.14 for pi.



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Find the surface area and volume.





## Answer Key

- |  |   |   |  |
|--|---|---|--|
| 1. -3  | 2. 237  | 3. -97  | 4. 196   |
| 5. 17  | 6. 339  | 7. 6  | 8. -41   |
| 9. 80  | 10. -81   | 11. 36  | 12. 26   |
| 13. 5  | 14. 97  | 15. -176  | 16. -120   |
| 17. 110  | 18. -15   | 19. -177  | 20. 14   |
| 21. -7   | 22. 98  | 23. 108   | 24. 34   |
| 25. 34   | 26. -27   | 27. -168  | 28. 77   |
| 29. -24  | 30. -55   | 31. -41   | 32. -16  |
| 33. 75.451   | 34. 0.525   | 35. 6.532   | 36. 131.2  |
| 37. 16.281   | 38. -11.5   | 39. 75.68   | 40. -100.2   |
| 41. 11.947   | 42. 1.68  | 43. 0.75  | 44. 62.464   |
| 45. $30\frac{3}{4}$                                  | 46. $1\frac{1}{20}$                                     | 47. $4\frac{1}{20}$                                       | 48. $2\frac{9}{20}$  |
| 49. $-1\frac{7}{9}$                                  | 50. -7  | 51. $-6\frac{5}{12}$                                      | 52. $\frac{5}{13}$   |
| 53. -2   | 54. $-14\frac{1}{5}$                                    | 55. $16\frac{1}{18}$                                      | 56. $-1\frac{7}{8}$  |
| 57. $j = -53$  | 58. $m = 39$  | 59. $x = -15$   | 60. $f = 18$   |
| 61. $g = -38$  | 62. $h = 117$   | 63. $b = -6$  | 64. $w = 70$   |
| 65. $m = 6$  | 66. $y = -20$   | 67. $r = -4$  | 68. $p = 22$   |
| 69. $k = -14$  | 70. $f = -5$  | 71. $g = -30$   | 72. $m = 2\frac{1}{2}$                                       |
| 73. $v = -28$  | 74. $n = 1$   | 75. $m = -2$  | 76. $y = 8$  |
| 77. $h = 5$  | 78. $c = 10$  | 79. $b = 28$  | 80. $j = 45$   |
| 81. $k = 6\frac{2}{3}$                               | 82. $a = 4$   | 83. $g = 7\frac{7}{8}$                                    | 84. $w = 4\frac{1}{2}$                                       |
| 85. 12.75  | 86. 8   | 87. 12.5%   | 88. 9  |
| 89. 20%  | 90. 27  | 91. \$46.40   | 92. \$14.98  |
| 93. $P = 34\text{ cm};$<br>$A = 48\text{ cm}^2$      | 94. $C = 72.22\text{ mm};$<br>$A = 415.265\text{ mm}^2$ | 95. $P = 22\text{ in};$<br>$A = 26\text{ in}^2$           | 96. $P = 63.7\text{ m};$<br>$A = 229.25\text{ m}^2$          |
| 97. $SA = 384\text{ cm}^2;$<br>$V = 512\text{ cm}^3$ | 98. $SA = 132\text{ in}^2;$<br>$V = 72\text{ in}^3$     | 99. $SA = 439.6\text{ cm}^2;$<br>$V = 461.58\text{ cm}^3$ | 100. $SA = 494.55\text{ mm}^2;$<br>$V = 826.605\text{ mm}^3$ |